1

Point estimators

Methods for finding estimators.
Properties of estimators.

Solve at least one of the following two exercises.

Exercise 1

Consider 10 independent and identically distributed observations from a Bernoulli family with success probability θ.

i) Write the likelihood function.

ii) Deduce the maximum likelihood estimator of θ.

iii) If the sample size n is equal to 3, write the likelihood function for each of the possible sampling results (x_1, x_2, x_3); $x_i = 0, 1$ and plot the corresponding function.

Exercise 2

Consider a Poisson distribution.

$$P(X = x) = \exp(-\theta)\theta^x/x!; \quad x = 0, 1, 2 \ldots$$

i) Compute E(X)

ii) Compute P(X=0)

iii) Let $Y = X^2$.
Compute P(Y=0)

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Test of hypotheses

Statistical hypothesis; test (non randomized) of a hypthesis; critical region; null and alternative hypothesis; simple and composite hypothesis; Type I and II errors; power function; size of type I and II errors.

Test of a simple null hypothesis against a simple composite hypothesis; most powerful test; Neyman-Pearson lemma.

Solve at least one of the two following exercises.

Exercise 1.

Consider a normal population with expectation θ and variance 1. Let $H_0 : \theta = \theta_0$ and $H_1 : \theta > \theta_0$.

i) Suggest a critical region for a size- α test.

ii) Compute the power function of the test.

Exercise 2

Let X be a random variable having a negative exponential distribution with parameter λ.

i) Write the density function.

ii) Compute $P(X > 0)$

iii) Compute $E(X)$

3

The linear model

Define the linear model; point estimators of the parameters; expected value and variance of the estimators; confidence interval and test of hypotheses for the regression coefficient.

Solve at least one of the following two exercises

Exercise 1

Consider a linear model with one independent variable. Assume normality of the errors with the zero expectation and constant variance σ^2.

i) Determine the (minimal) sufficient statistics.

ii) Suggest an unbiased estimator for the variance σ^2; *show* that it is unbiased.

Exercise 2.
Let X be a random variable uniformly distributed over the interval $(0, 1)$.

i) Write the cumulative distribution function of X.

ii) Let $Y = g(X) = X^2$. Find the density of Y.