Dipartimento di Statistica e Metodi Quantitativi

Multidimensional scaling of asymmetric relationships

Akinori Okada

Professor Emeritus of the Rikkyo University, Tokyo
okada@rikkyo.ac.jp

November 21, 2019 - h. 11.00 Seminar Room: 4026
Building U7 – 4th floor

Torgerson (1952) introduces his multidimensional scaling as the first practical method of representing proximity (similarity, dissimilarity) relationships among objects in a multidimensional space or a configuration. The method is called metric, which means that the observed proximities are assumed to be interval or ratio scaled values. Shepard (1962a, b) and Kruskal (1964a, b) introduced methods which are called nonmetric multidimensional scaling. Nonmetric multidimensional scaling assumes that the observed proximities are ordinal scaled values. This mean that only the information which of two proximities is larger (or smaller) than the other is utilized in the analysis. The nonmetric multidimensional scaling of Kruskal (1964a, b) has been used most frequently in practical applications of multidimensional scaling, and several multidimensional scaling methods based on the similar idea of Kruskal (1964a, b) have been introduced. The multidimensional scaling developed so far assumes that the proximity is symmetric.

Several researchers focused their attention on the asymmetry in various sorts of proximity; similarity/dissimilarity judgment, same/different judgment, reaction time, social and psychological distance, and so on. It is meaningful to develop asymmetric multidimensional scaling which can represent asymmetric proximity relationships among objects in a configuration. Various kinds of procedures to analyze an asymmetric proximity matrix were invented. We concentrate on the multidimensional scaling which is based on the distance among points, each representing an object, in a multidimensional space (distance model).

The asymmetric multidimensional scaling by Okada and Imaizumi (1987) represents asymmetric proximity relationships among objects by the interpoint distance in a multidimensional configuration. The distance between two points is symmetric. It is difficult to represent asymmetric relationships simply by the interpoint distance, because the distance itself is symmetric. The model of Okada and Imaizumi (1987), which is called Distance-radius model, represents an object by a point and a circle [two-dimensional model] (sphere [three-dimensional] or hypersphere [larger than three-dimensional]) centered at the point in a multidimensional space. Several models were derived from the model.

Multidimensional scaling mentioned above, regardless of metric or nonmetric and of being able to represent asymmetry or not, analyzes one proximity matrix (object x object) or one-mode two-way proximities (Carroll and Arabie, 1980). To analyze a set of proximity matrices each comes from a source (object x object x source) or two-mode three-way proximities, INDSCAL was developed by Carroll and Chang (1970). INDSCAL can represent relationships among objects and how each source (individual) has different relationships among objects each other. While INDSCAL can represents differences among sources, it cannot represent the asymmetry in relationships among objects of each source. The model of Okada and Imaizumi (1997) is an extension of the Distance-radius model which can analyze a set of asymmetric proximity matrices or two-mode three-way asymmetric proximities. The asymmetric multidimensional scaling of Okada and Imaizumi

(1997) can represents asymmetric relationships among objects, and how each source has different symmetric and asymmetric relationships among objects each other as well. Several models, which are less restricted or more restricted, have been derived from Okada and Imaizumi (1997).

To analyze one-mode two-way asymmetric proximities, there is another asymmetric multidimensional scaling by Okada (2011) and Okada and Tsurumi (2012) which is based on the singular value decomposition. The asymmetric multidimensional scaling has been utilized to analyze mainly the data from marketing research or brand switching. Several models and procedures have been derived by extending the model of Okada (2011) which can, for example, evaluate newly introduced brands, diagnose brands, tell how to increase the dominance of a brand in the brand switching... Another sort of procedure to analyze one-mode two-way asymmetric proximities is asymmetric cluster analysis, which can represent asymmetric proximity relationships among objects by forming clusters (Okada & Iwamoto, 1966; Okada & Yokoyama, 2015). It is important and meaningful to develop not only multidimensional scaling but also cluster analysis as well so that we can use two different kinds of methods to analyze the same proximity data (Arabie & Hubert, 1994).

There are quite a few number of applications of asymmetric multidimensional scaling and cluster analysis. While these applications are not mentioned here because of the space limitation, some of them will be mentioned in the talk. The present abstract deals only with researches of the author and his colleagues. Bove and Okada (2018) gives a review which covers a wide range of asymmetric multidimensional scaling and asymmetric cluster analysis in detail. Some books mention the analysis of asymmetric relationships. Borg and Groenen (2005) has a chapter of asymmetric multidimensional scaling. Coxon and Coxon (2001) refers to asymmetric multidimensional scaling.

References

Arabie, P. & Hubert, L. (1994). Cluster analysis in marketing research. In Bagozzi, R. F. (Ed.), A*dvanced methods marketing research*. pp. 160-199. Cambridge, MA: Blackwell.

Bove, G., & Okada, A. (2018). Methods for the analysis of asymmetric pairwise relationships. Advances in Data Analysis and Classification, **12**, 5-31.

Borg, I., & Groenen, P.J.F. (2005). *Modern multidimensional scaling: Theory and applications* (2nd ed.). New York: Springer.

Carroll, J. D., & Arabie, P. (1980). Multidimensional Scaling. In M. R. Rosenzweig & L. W. Porter (Eds.), *Annual Review of Psychology*, **31**, 607-649.

Carroll, J. D. & Chang, J. J. (1970). Analysis of individual differences individual differences in multidimensional scaling via an *N*-way generalization of "Eckart-Young" decomposition. *Psychometrika*, *35*, 283-319.

Coxon, T.F., & Coxon, M.A.A. (2001). *Multidimensional scaling* (2nd ed.). Boca Raton, FLChapman&Hall/CRC.

Kruskal, J. B. (1964a). Nonmetric multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika*, *29*, 1-27.

Kruskal, J. B. (1964b). Nonmetric multidimensional scaling: A numerical method. *Psychometrika*, **29,** 115-129.

Okada, A. (2011). Analyzing social affinity for foreigners by spectral decomposition: Asymmetric multidimensional scaling of EASS 2008 Data [Supekutoru bunkai niyoru gaikokujin ni taisuru shinkinkan no bunseki: EASS 2008 no deta wo mochiita hitaishou tajigenshakudokoueihou no ouyou]. In JGSS Research Center, (Ed.), *JGSS Research Series*, *11*. (pp. 119-128). Osaka: Osaka University of Commerce. (in Japanese) available at http://jgss.daishodai.ac.jp/research/monographs/jgssm11/jgssm11_11.pdf

Okada, A., & Imaizumi, T. (1987). Nonmetric multidimensional scaling of asymmetric proximities. *Behaviormetrika*, **21**, 81-96.

Okada, A., & Imaizumi, T. (1997). Asymmetric multidimensional scaling of two-mode three-way proximities. *Journal of Classification*, **14,** 195-224.

Okada, A., & Iwamoto, T. (1996). University enrollment flow among the Japanese prefectures: A comparison before and after the Joint First Stage Achievement Test by asymmetric cluster analysis. *Behaviormetrika*, **23**, 169-185.

Okada, A., & Tsurumi, H. (2012). Asymmetric multidimensional scaling of brand switching among margarine brands. *Behaviormetrika*, **39**, 111-126.

Okada, A., & Yokoyama, S. (2015). Asymmetric CLUster analysis based on SKEW-Symmetry: ACLUSKEW. In I. Morlini, M. Tommaso, & M. Vichi, (Eds.), *Advance in statistical models for data analysis*. 191-199. Heidelberg, Germany: Springer-Verlag.

Shepard, R. N. (1963a). The analysis of proximities: Multidimensional scaling with unknown distance function. I. *Psychometrika*, **27**, 125-140.

Shepard, R. N. (1963b). The analysis of proximities: Multidimensional scaling with unknown distance function. II. *Psychometrika*, **27,** 219-246.

Torgerson, W. T. (1952). Multidimensional scaling: I. Theory and method. *Psychometrika*, **17**, 401-419.

For more info: ***fulvia.pennoni@unimib.it***